A 50—I

OXFORD LOCAL EXAMINATIONS

General Certificate of Education

Summer Examination, 1964

Advanced Level

PURE AND APPLIED MATHEMATICS, PAPER I

Tuesday, 16 June. Time allowed: 3 hours

Write the number of the paper, **A 50/I,** *on the left at the head of each sheet of your answers in the space provided.*

Complete answers to a few questions receive more credit than scrappy attempts at the whole paper. A pass may be obtained on good answers to about **four** *questions.*

Mathematical tables and squared paper are provided.

1. Expand $(1+x)^{-\frac{1}{2}}$ by the binomial expansion, giving the first three terms and the general term.

Prove that

$$\sqrt{5} = \frac{9}{4}\left\{1 - \frac{1}{2}\cdot\frac{1}{80} + \frac{1.3}{2.4}\left(\frac{1}{80}\right)^2 - \ldots\right\},$$

and deduce that $\sqrt{5}$ lies between 2·2359 and 2·2361.

2. Prove that for real values of x the value of the function $\dfrac{4x+5}{2x^2+4x+3}$ cannot be greater than 2 or less than -1, and find the values of x for which it takes the values 2 and -1.

3. (a) Express

$$\frac{1}{(1+rx)\{1+(r-1)x\}}$$

in partial fractions and find the sum of the first n terms of the series of which this is the rth term.

(b) The value of a certain physical quantity is given by the expression $4{\cdot}3-1{\cdot}6e^{-t^2/51}$, where t is the time from a certain instant. Given that $\log_{10} e = 0{\cdot}4343$, use the tables provided to calculate the value of t when this expression is equal to $3{\cdot}5$.

4. Solve the following equations, giving, in each case, the general solution:

(a) $3\cos\theta-\sin\theta = 1$,

(b) $2\cos^2\theta-\sin\theta\cos\theta-\sin^2\theta = 1$.

5. The length of the perpendicular from the vertex A of triangle ABC to BC is p and the circumradius of the triangle is R. Prove that

$$\cos(B-C) = \frac{p}{R}-\cos A.$$

Given p, R and A, show how to calculate B and C, and show that, if $R = 2p$ and $A = 60°$, then $C = 15°$ or $105°$.

6. The angle of elevation of a peak from an observation post A, south-west of it, is $70°\ 32'$. Its angle of elevation from a second post B, due south of it, is $47°\ 21'$. The post B is 1500 ft. above A and its bearing from A is $112\frac{1}{2}°$ (i.e. $22\frac{1}{2}°$ south of east). Find, to the nearest 10 ft., the height of the peak above A.

7. The horizontal base ABC of a triangular pyramid $VABC$ is an equilateral triangle of side 2 in. The edges VA, VB and VC are each 3 in. in length. P and Q are the mid-points of VB and VC. The pyramid is cut by a vertical plane through PQ. Draw, to a suitable scale, the cross-section of the pyramid by this plane and calculate the lengths of its sides.

8. Find the condition that the two lines $y = m_1 x$, $y = m_2 x$ shall be at right angles.

Two lines through the origin, O, make angles, measured in the same sense, of $\alpha°$ and $90°-\alpha°$ with the positive direction of the x-axis. They meet the line $x = a$ at the points P and Q. The line through P at right angles to OP meets the line through Q at right angles to OQ at the point R. Find the locus of the point R as α varies.

9. Prove that the line $y = mx + \sqrt{(a^2m^2+b^2)}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, and find the coordinates of its point of contact.

Find the equation of that common tangent to the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2} = 1$ whose points of contact, T and T', lie in the first quadrant. If O is the origin and P is the point of intersection of these ellipses in the first quadrant and if OP meets the common tangent TT' at Q, prove that $OP.OQ = ab$.

10. Prove that the equation of the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is $y + tx = 2at + at^3$.

If P and Q are the extremities of a chord of the parabola $y^2 = 4ax$ through its focus, prove that the normals at P and Q intersect on the parabola $y^2 = a(x-3a)$.