O 50—II

OXFORD LOCAL EXAMINATIONS

General Certificate of Education

Summer Examination, 1964

Ordinary Level

GEOMETRY

Wednesday, 1 July. Time allowed: $2\frac{1}{2}$ hours

Write the number of the paper, **O 50/II,** *on the left at the head of each sheet of your answers in the space provided.*

Answer **Questions 1, 2, 3, 4,** *and any* **two** *of* **Questions 5, 6, 7,** *and* **8.**

No credit will be given for any attempt at a question in Practical Geometry if any of the construction lines are erased. When parallels or perpendiculars are drawn, the method used must be stated.

The use of a straight edge or compasses must, in all cases, be indicated by a drawn straight line or arc; that is, any constructed point must be shown as the intersection of two lines, not as a dot on a line. Do not draw exact figures unless these are necessary. Mathematical tables are provided.

1. Prove that the diagonals of a quadrilateral whose sides are all equal bisect its angles.

$ABCD$ is a rhombus in which angle ABC is 120°. The diagonal AC is produced to a point F. The bisector of angle ABD meets AD in K and KB is produced to E so that $BE = CF$. Prove that

(i) triangles ABD and DBC are equilateral,

(ii) angles EBD and FCD are each 150°,

(iii) triangles BDE and CDF are congruent,

(iv) angle $EDF = 60°$,

(v) triangle DEF is equilateral.

2. Without assuming any formula for the area of a triangle, prove that triangles on the same base and between the same parallels are equal in area.

E is a point on the side DC of the parallelogram $ABCD$ and BE produced meets AD produced at F. Prove that triangles AEF and BDF are equal in area and that triangles AED and CEF are equal in area.

3. ABC is a triangle with AC greater than AB. The perpendicular bisector of BC meets AC at D. Prove that $AD+DB = AC$ and that the angle ADB is twice the angle ACB.

Use these results to construct a triangle ABD in which $AB = 2$ in., $\angle ADB = 64°$, $\angle DAB$ is acute and the sum of the sides AD, DB is 3 in. State briefly the main steps in your construction.

Calculate $\angle DAB$ and check your result by measurement.

4. Prove that opposite angles of a quadrilateral inscribed in a circle are supplementary.

$ABCD$ and $ABEF$ are two circles intersecting at A and B, and DAF and CBE are parallel straight lines such that A is between D and F and B is between C and E. Prove that $DF = CE$.

Not more than **two** *of* **Questions 5, 6, 7, 8** *are to be attempted*

5. Prove that, if a straight line AB subtends equal angles at two points C, D on the same side of it, then the four points $ABCD$ lie on a circle.

AQP are three points on a straight line, in that order. Two circles are drawn, one through A and Q and the other through A and P. These circles meet again at B, and O is the centre of the circle ABP. The line BQ cuts circle APB at R. Given that AR is parallel to BP, prove that O lies on circle AQB.

6. Prove that similar triangles have areas in the ratio of the squares on corresponding sides.

The triangle *BAC* is right-angled at *A* and the lengths of *BC*, *CA*, *AB* are a, b, c respectively. The line perpendicular to *BC* at *B* meets *CA* produced at *E* and the line perpendicular to *BC* at *C* meets *BA* produced at *F*. Prove that

(i) triangles *EAB*, *ABC*, *ACF* are similar,

(ii) triangle $EAF =$ triangle *BAC* in area,

(iii) $\dfrac{\text{area of quadrilateral } EFCB}{\text{area of triangle } ABC} = \dfrac{a^4}{b^2c^2}$.

7. Two buoys, *A* and *B*, floating on the sea, are known to be a quarter of a mile apart. A man at *P*, on a cliff, measures their angles of depression to be 30° and 45° and the angle *APB* to be 60°. Calculate the height of *P* above sea level to the nearest foot.

8. *OABCD* is a pyramid whose base *ABCD* is a square of side 2 in. and whose faces *OAB*, *OBC*, *OCD* and *ODA* are equilateral triangles. *P*, *Q*, *R* and *S* are points in *OA*, *OB*, *OC* and *OD* respectively, such that $OP = OQ = \frac{1}{2}$ in. and $OR = OS = 1\frac{1}{2}$ in. Draw the section of the pyramid by the plane *PQRS* and, by making any necessary measurements or otherwise, find its area.